The Existence, Transcendence, and Evolution of the Subject—A Method Based on Subject Information

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Abstract—**This paper details a new laboratory project in a senior-level Mechanical Engineering Vibrations course. Students are to determine the first four natural frequencies of a 6061 Aluminum free-free beam in a laboratory using three methods: (1) idealized continuous beam model (2) Finite Element Analysis (FEA), and (3) experimentally. Using student survey data, it is shown that the project bolstered the following skills: (A) use of measurement equipment to acquire and transmit real-world data, (B) performing a Discrete Fourier Transform (DFT) and creating the Power Spectral Density (PSD) plot of empirical data, (C) creating and modifying FEA code in Matlab to find natural frequencies and test for convergence of results, and (D) connecting the distinct topics of the course together.**

Index Terms—**Arduino, project, vibrations**

I. Introduction

OUR Mechanical Engineering program's Mechanical Vibrations course has been completely based in theory and simulation. Students did not have hands-on interaction with a real-life spring, mass or damper nor a beam in oscillation. Students may take a dynamics lab as a laboratory elective but can graduate without practical application of the vibrations topics. Experiential and real-world application of theory reinforces what is learned in the classroom [1], [2]. This hands-on method of learning is beneficial to students with preferred learning styles other than lecture [3], [4].

Students typically take a Matlab programming course during their first or second year but do not continually apply it in their later courses. Furthermore, most of their programming experience consists of solving problems from a Matlab textbook and they do not have a chance to analyze their own experimental data.

Our students have the option to take a microcontrollers class as an elective, but again, can graduate without interfacing a computer to the real world through sensors. This interfacing has been shown to increase students' ability at programming [5]. A microcontroller gaining popularity because of its low cost and flexibility is the Arduino. Students are able to quickly create the hardware and software for their Arduino project [6]. Additionally, the community support and tutorials help the students with their project [7]. Previous work has used Arduinos in Vibrations courses for collecting experimental data [8]. For demonstrating and educational purposes, the results of the data collection were comparable to analytic results.

For an engineer, the ability to interface computer programming with their engineering knowledge is an important skill for their future work [9]–[11]. The ability to analyze real-world data has been seen as an important ABET outcome by industry [9], [10] and technical skills [11].

The intention of this project was to act as a comprehensive assignment that combined what students learned in this Mechanical Vibrations course with what they have learned previously in their Instrumentation, Matlab programming, and technical writing courses. Students were given the chance to collect real data on a physical object and created a program to analyze the empirical data. They then compared their actual results to what the theory predicted. The sensor and microcontrollers cost only a few dollars and the implementation was fast. This project addition was made in the Fall 2016 and 2017 versions of the course. An early version of this work with only 2016 survey data was presented at ASEE 2017 [12].

II. Project Description

Students were asked to determine the first four natural frequencies (in transverse vibration) of a 6061 Aluminum free-free bar (Fig. 1) utilizing three methods. First, they were to calculate the values based on the theoretical Euler-Bernoulli 4^{th}-order partial differential equation (PDE). Next, they were to create a finite element code and determine the frequencies numerically. Finally, they were to determine them experimentally by recording and analyzing acceleration data. Using the natural frequencies found from the three methods, they then compared the results.

Students first measured the dimensions of the beam (Table I) and found the appropriate material properties for 6061 Aluminum. Nominal values for the Modulus of Elasticity and material density were used. For the analytic method [13], students were expected to derive the frequencies by beginning with the 4^{th}-order PDE for a beam in transverse vibration. Applying the four boundary conditions (zero shear force and bending moment at the ends), they found the eigenvalues. The first four eigenvalues were used with the material properties and dimensions to calculate the first four natural frequencies. This was done for vibrations along both principal transverse axes.

Students were provided a basic beam FEA code for modal analyses of beams in transverse deflection [14]. They needed to modify the code for use with the specific beam and apply a mesh. Then, the local and global stiffness and mass matrices were created (it was assumed to have negligible damping). Then, they applied the proper boundary conditions for these

Fig. 1: Aluminum Beam Suspended from Ladder. The “T”-top of the beam was hung loosely with a loop of rope on each side attached to an eyehook at the top of the ladder. The bottom of the beam was suspended above the ground. An ADXL-335 accelerometer was affixed to the bottom of the beam and shown in more detail in Fig. 2.

TABLE I: BEAM DIMENSIONS

Length	51”
Width (x-axis)	1.25”
Height (z-axis)	1”
Beam Weight	6.6 lb
Accelerometer Weight	0.04 oz

supports. No modification to the matrices was needed for this free-free boundary condition. The matrices were used to find the eigenvalues of the system. Finally, the natural frequencies were calculated using the eigenvalues. Using the FEA code, they needed to demonstrate the convergence of their solutions by solving with an increasing number of elements (thus increasing the resolution of the mesh). Again, this process was done for vibrations along both principal transverse axes.

In the experimental section, students utilized an ADXL-335 analog accelerometer affixed to the bottom end of the beam (Fig. 2). The accelerometer was wired to the Arduino microcontroller as shown in Fig 3. Students used either an Arduino Nano or Uno for this project and were provided a template Arduino code [15] for reading the sensor and logging it to the serial terminal. They needed to modify the code to poll at an appropriate sample rate and read from the correct analog pins on the microcontroller.

Students began the data logging to record acceleration along the x- or z-axis. Then, they used the sharp tip of a welding hammer to strike the beam (treated as an impulse) along that axis and allowed it to vibrate for at least 90 seconds. After stopping the data logging, the time and acceleration data were copied out of the terminal and into a spreadsheet for later analysis. This data collection was repeated several times along the same surface, striking at different locations. Then, the experiment was repeated along the other principal transverse axis.

The raw data were imported into Matlab one trial at a time. The Arduino did not sample uniformly so data had to first be resampled, using linear interpolation, to a uniform

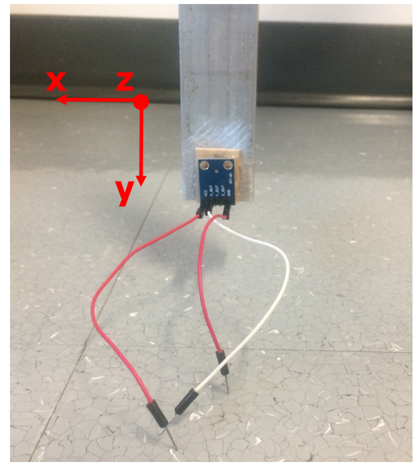

Fig. 2: ADXL-335 Accelerometer Affixed to Beam End. The accelerometer was hot-glued to a small piece of wood which was hot-glued to the beam. This is treated as a rigid connection to the beam. The positive sense of the sensor axes is indicated by the inset coordinate system.

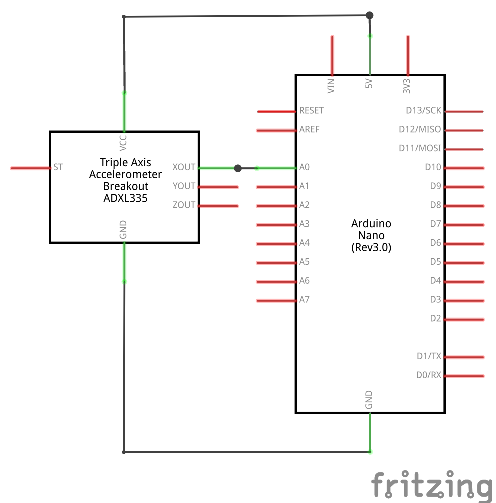

Fig. 3: Wiring Diagram. In this schematic, the x-axis acceleration is being measured. For the z-axis test, the wire at *XOUT* was moved to *ZOUT*

time step. The uniform time was set to the average sample rate, typically about 1800 Hz. Students plotted the data and decided where to trim to include only those data after the initial strike and before the vibrations become “too small.” A Discrete Fourier Transform (DFT) was performed on each truncated data set. The coefficients were divided by the square of their associated frequency (effectively integrating) to find the coefficients for the position of the end of the beam. These coefficients were used to create a Power Spectral Density (PSD) plot. Students manually identified the first four peaks in the PSD and recorded these as the natural frequencies, again assuming no damping in the system. Values from the multiple trials were averaged together to find the experimental values.

III. Sample Student Work

Using the dynamic beam equation [13], the measured dimensions of the bar and the material properties, students first

TABLE II:
ANALYTICALLY-DETERMINED NATURAL FREQUENCIES

ω_n	x-axis (rad/s)	z-axis (rad/s)
1	617	494
2	1702	1361
3	3336	2669
4	5514	4412

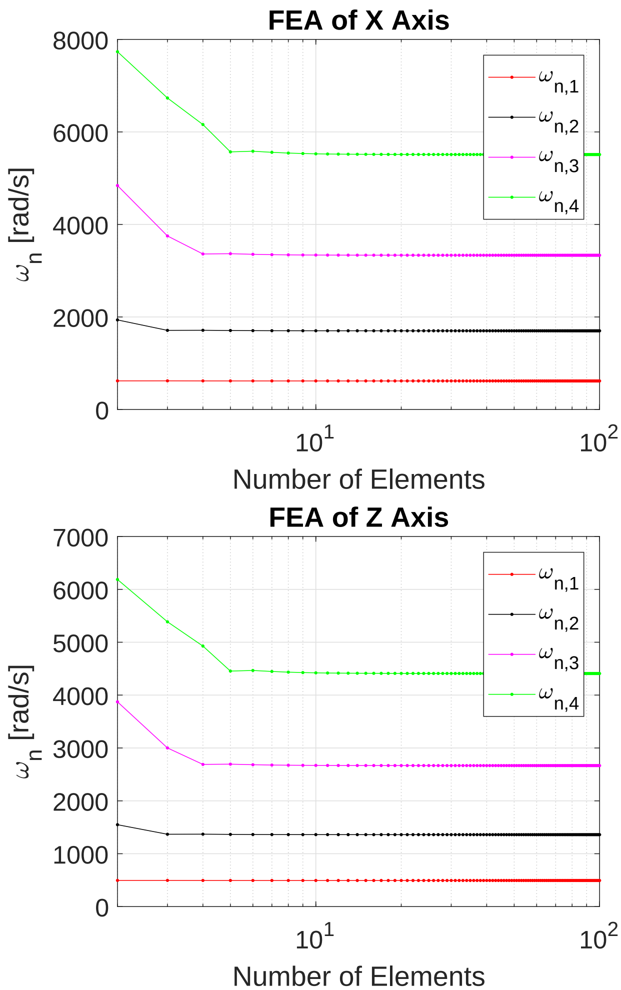

Fig. 4: Convergence Plots for FEA-determined Natural Frequencies. The first four natural frequencies of the beam converged by six elements for both principal axes. The converged values are listed in Tables III and IV.

found the natural frequencies for principal axes x and z as shown in Table II.

By modifying provided FEA code, the numerically-determined natural frequencies were calculated. These values differed by less than 0.1% from those found by the analytic method. Because the weight of the accelerometer was much smaller than the weight of the beam ($< 0.04\%$ of the beam weight), it was not accounted for in the FEA analyses. A typical plot of the convergence of the natural frequency values as the number of elements is increased is shown in Fig. 4. For this simple beam, convergence to the analytic solution for the first four frequencies was reached by five elements.

Students ran the experimental protocol, outlined in Sec. II, multiple times for transverse vibrations about the x- and z-axes. Collected acceleration values were kept in bits (0-

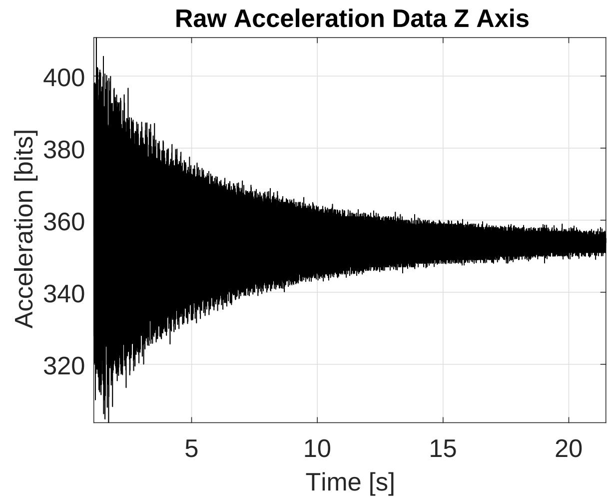

Fig. 5: Typical raw acceleration data. Data output from the Arduino was kept in bits as only the spectral information was desired.

TABLE III:
TYPICAL COMPARISON OF NATURAL FREQUENCIES (RAD/S) FOR x-AXIS

ω_n	Analytical	FEA	Experimental	Difference
1	617	617	555.4	10%
2	1702	1702	1532	10%
3	3336	3336	2946	11.7%
4	5514	5514	4033	26.9%

1023), rather than g's, from the 10-bit Arduino analog-to-digital conversion. For this project, the scale of the acceleration values does not impact the determination of the natural frequencies from the PSD. Leaving the data in bits removed the need to calibrate the accelerometer. A typical plot of the trimmed acceleration data is shown in Fig. 5. The magnitude of acceleration is clearly attenuated over the time span of recording. For this project, this damping is ignored as it was in the analytical and numerical sections.

After performing a DFT on the acceleration data and integrating twice, the coefficients of the *position* were found. Typical PSD plots using these coefficients are shown in Fig. 6. The power at the n^{th} frequency was defined as

$$P_n = \sqrt{a_n^2 + b_n^2}, \tag{1}$$

where a_n and b_n are the n^{th} coefficients from the DFT. Students manually determined the first four peaks of the PSD as marked in the figures. Students were not to use built-in functions such as `pwelch` for this assignment.

Comparisons of the first four calculated natural frequencies for the x- and z-axes in this sample are shown in Table III and IV respectively. The experimental values were averaged from the PSDs from three or four trials. Frequencies determined by the Arduino and PSD analysis were generally "close" to the analytic and numeric values. In most cases, all four identified frequencies were lower for the experimental values. These typical results from the experimental portion are consistent with those found in previous classroom experiments using an Arduino to measure natural frequencies [7].

In analyzing the sources of the discrepancies between calculated and measured values, students concluded that the

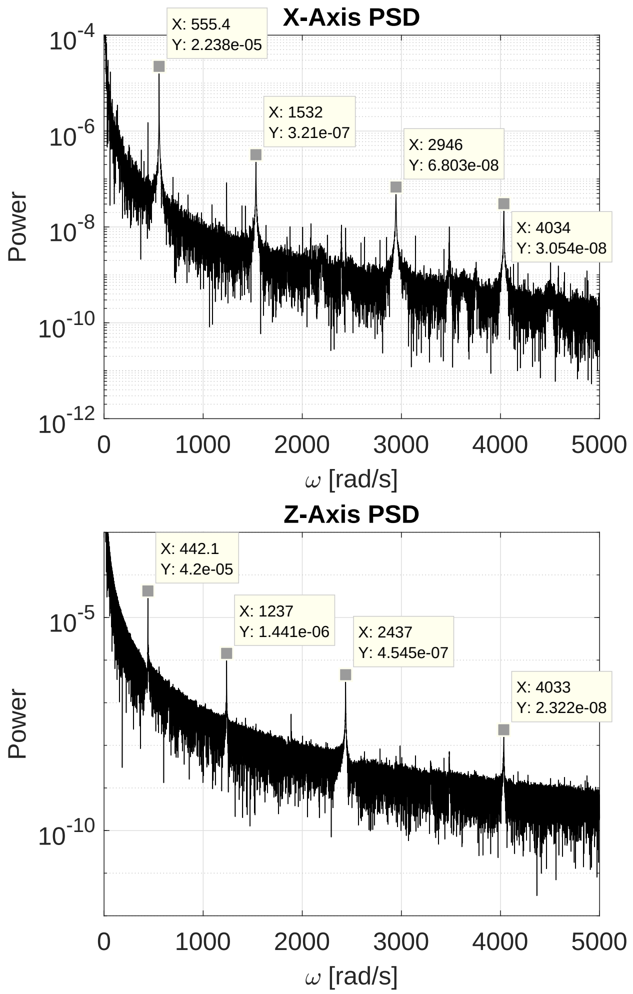

Fig. 6: PSD Plots. Four clearly identifiable peaks were found on each the plot for each axis. The power is calculated based on Eq. (1).

TABLE IV:
TYPICAL COMPARISON OF NATURAL FREQUENCIES (RAD/S) FOR z-AXIS

ω_n	Analytical	FEA	Experimental	Difference
1	494	494	442.1	10.5%
2	1361	1361	1237	9.1%
3	2669	2669	2437	8.7%
4	4412	4412	4033	8.6%

assumption of zero damping was not completely accurate. The attenuation in the acceleration plot (Fig. 5) and the difference in natural frequencies demonstrated the limitation of this assumption on real-world problems.

The nominal density value for 6061 Aluminum was used for the analytical and numerical determination of the natural frequencies. However, when using the measured dimensions and weight the calculated density was about 6% higher. This calculation includes the weight of the T-bar at the top of the beam. Accounting for this higher density, the analytically- and numerically-determined natural frequencies reduce by 3%.

The accelerometer was not mounted to perfectly coincide with the principal axes of the beam. Therefore, the acceleration measurements contained information from a slightly oblique axis to the bar.

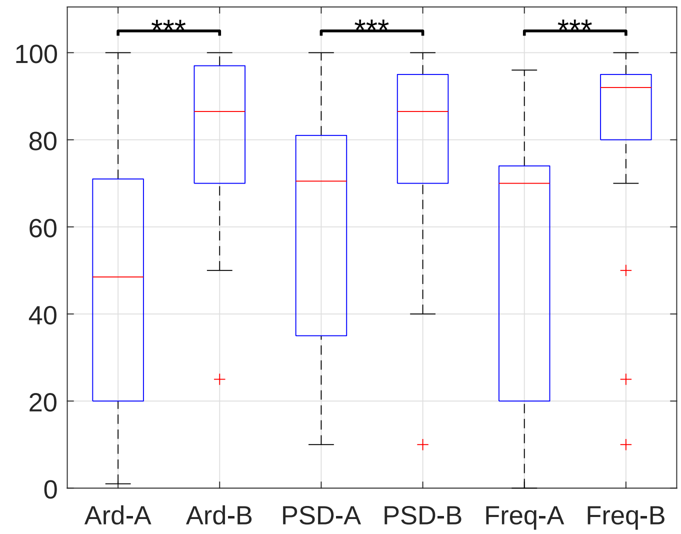

Fig. 7: Boxplot of Survey Results (1). There is a significant difference between the median pre-project ability (A) and the post-project ability (B) for all prompts. Two stars indicates p-value of $p < 0.002$ and three stars for $p < 0.0002$. Outliers are marked as "+" but are included in all statistics.

Finally, the boundary conditions at the top and bottom of the beam are not truly "free-free." There is rope attached at the top and the accelerometer at the bottom of the beam is connected by wires to the Arduino. These connections certainly impact and dampen the motion of the beam.

The observed differences in experimental values versus the idealized theoretical values demonstrated the difficulties of modeling an ideal system versus how it behaves in reality [2]. Students were able to see first-hand that the two do not agree and that they need to think about possible sources of the discrepancies. It is important that the students realize that the models created in the classroom may not be appropriate to apply on actual systems they encounter in the field.

IV. Student Outcome Measurements and Discussion

After submitting their final project, students were asked to complete a survey about the project. The survey was administered at the end of the Fall 2016 and 2017 semesters. The responses were pooled. Students were asked to rate their ability in several areas on a 0-100 continuous scale before starting and after finishing the project. Statements were anchored at 0, 33, 66 and 100 for each of the prompts but students could select any integer value. The prompts and anchor statements are shown in Table V. The code in italics under each prompt corresponds to the responses in Figs. 7 and 8.

The survey was distributed via Qualtrics (Provo, UT) at the end of each semester and was completed by $n = 22$ students. Between two semesters, 24 groups completed the project. The results of the numerical responses are displayed as boxplots in Fig. 7 and Fig. 8. The prompts are in the same order as presented in Table V. The "A" response is perceived ability before starting the project while "B" is perceived ability after finishing the project. All statistical analyses were performed in Matlab (Natick, MA, R2017a).

TABLE V:
PROMPTS AND ANCHORS FOR STUDENT SURVEY

(*Code*) Prompt	0	33	66	100
(*Ard*) Rate your proficiency level of wiring and programming an Arduino (or PIC) for data collection...	I have never attempted this	I know how to read and create very basic code but cannot attach a sensor	I am able to attach the sensor to the Arduino but not read data from it properly	I can read data taken from a sensor I have attached to the Arduino in a format ready to be used in a subsequent step
(*PSD*) Rate your proficiency level of performing a Discrete Fourier Transforms (DFT) and creating and interpreting Power Spectral Density (PSD)...	I do not know what either of these is	I can find the Fourier coefficients but am not sure what to do with them	I can make a PSD plot using my coefficients, but I am not sure what it shows me	I am able to find the Fourier coefficients, plot them on the PSD and then identify peaks on the PSD
(*Freq*) Rate your proficiency level of finding natural frequencies of continuous beams using analytic methods...	I do not even know where to start	I can identify the boundary conditions correctly	I can set up the equation, but not fully solve for the natural frequencies	I can set up the equations and solve for the natural frequencies numerically
(*FEA*) Rate your proficiency level of creating (or modifying) Finite Element Analysis (FEA) code to find natural frequencies of continuous beams...	I do not even know where to start	I can set up some things (e.g. parameters of the system and boundary conditions) but the code will not run	I can run the code, but the results do not look correct	I can size my elements correctly, create my mass and stiffness matrices, apply proper boundary conditions and then find the eigenvalues
(*Content*) Rate your level of overall understanding of the course content and how it fits together...	I do not understand anything in this course	I can generally identify the type of system I have but I am not always sure what equations are appropriate to use	I am able to identify the type of system I have and find the appropriate equations to solve for unknowns, but I am not sure how the units all fit together	I am able to identify the type of system I have and find the appropriate equations to solve for unknowns. I also understand how all the mathematics review, MDOF systems, and continuous systems fit together.

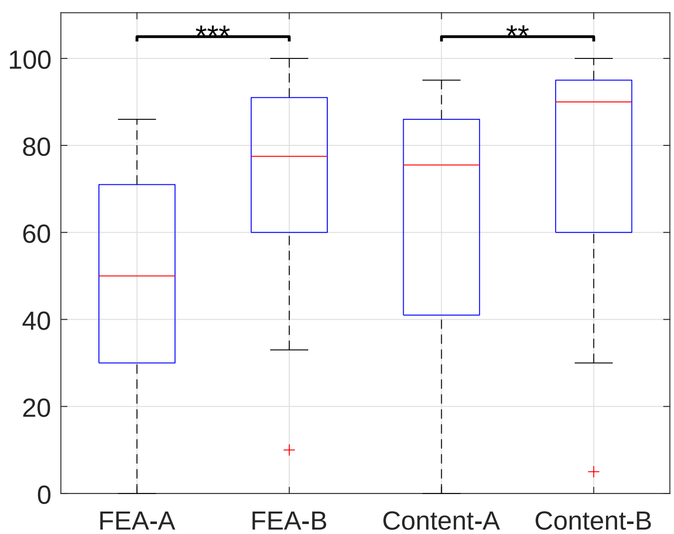

Fig. 8: Boxplot of Survey Results (2). There is a significant difference between the median pre-project ability (A) and the post-project ability (B) for all prompts.

Distributions were not normally or symmetrically distributed around the median. Additionally, the sample size was small ($n < 30$). Therefore, to compare the median pre- and post-ability for each prompt, a one-sided Sign test [16] was employed. All median post-measurements (B) were found to be significantly larger than the pre-measurements (A) with 95% confidence, i.e. $p < 0.01$ using a correction factor for the five tested categories.

Even though there was a significant increase in perceived ability across the five domains, this should be expected as students should not *lose* ability after completing a project. To further assess how much each domain was improved, the effect size between the pre- and post was evaluated. Due to the small sample size and matched evaluations, Hedge's g_{av} [17], [18] was used to calculate the effect size. The calculated values are shown in Table VI. All domains showed a "large" effect size except for *Content* which showed a "medium' effect size [19].

TABLE VI:
HEDGE'S g_{av} VALUES FOR SURVEY RESPONSES

Ard	PSD	Freq	FEA	Content
1.0927	0.7328	1.0529	1.0138	0.5312

One limitation to the following inferences of the survey results is that students completed these projects as a team so not every member may have been as involved in every aspect of the project. Additionally, all ability assessments were self-reported and an objective concept quiz was not administered before or after the project.

A gain in the median ability to use microcontrollers was observed. In the pre-ability, we see a large spread of experience with Arduinos. Several of the students were concurrently taking an elective microcontrollers class using PICs and were already familiar with using sensors and controllers. We observe this spread decrease in the post-ability. This domain had the largest g_{av} value, showing a marked increase in student familiarity and ability with using the Arduino.

The students' ability to create and interpret PSDs was greatly increased after completing the project. At the beginning of the course, we discuss and create PSDs as part of learning about Fourier Transforms and DFTs. However, at that point in the course, the discussion is limited to 1DOF of systems and using premade empirical data sets.

Students' ability to analytically and numerically determine natural frequencies was improved. Because continuous systems is the last topic in the course, they had the least experience with these skills before starting the project. They completed homework assignments on the topics, but this was the first time they had a chance to use them on a larger scale.

Finally, there is an increase in the students' understanding of how the topics in the course fit together. Topics are initially presented in silos: mathematics, MDOF systems, and continuous systems. This project was the first assignment to synthesize topics from the entire semester and have students apply them to solve a real-world problem. However, this domain had the lowest g_{av}. The connections between topics need to be made more explicit in the next version of the assignment.

Anecdotally, students reported in the survey that the hands-on project helped their understanding of the theory presented in class. This report agrees with previous literature on the benefits of laboratory activity [3]. Additionally, students appreciated working with teammates with different strengths. The team members were able to learn from each other while working on the project.

The positive results from the survey data demonstrate that the project was worthwhile and met the intended objectives of improving student ability in several areas. Students were asked in the survey to list what they found most frustrating in completing the project and suggestions to improve it. These responses will be taken into account to improve the project for next year's class. Additionally, an objective measurement will be made to further gauge the improvement of student ability.

V. Conclusions & Future Considerations

Based on student feedback from the survey and grading the submitted projects, several changes are planned for the next version of the project. One issue was that the project was not assigned until the end of the course when all the topics had been covered. This did not allow enough time to thoroughly complete the project and complete a report. In the future, the project will be assigned earlier as the data collection and FEA can be done before covering the final course topic of continuous systems.

The scope of the project will be extended in the next version. The natural frequencies along the axial direction (y-axis) can also be assessed. Adding axial vibrations will require a different analytical and numeric approach than the two transverse axes. However, the natural frequencies are much higher axially so either a faster-sampling micro-controller or a larger bar will be required. Additionally, the end conditions can be altered to fixed or pinned to change the natural frequencies. This too will likely need a higher sample rate. Finally, the damping effects in the experimental data can be estimated using the logarithmic decrement. This may yield a better estimate of the natural frequencies by first calculating a damping ratio.

Students appreciated using the Arduino and the sensor but felt the code template gave them too much at the start of the project. They would have preferred coding the Arduino from a blank template and understanding how it worked better. In the next version, students will be given coding resources and references but will complete the hardware and software interfacing themselves. With the experimental portion beginning earlier in the semester, they will have more time to do focus on the FEA and continuous analysis at the end of the semester.

The main purpose of the project was to expose students to a real-world application of vibrations analysis. The quality of the data was not superb but with a few modifications, could be improved. First, as shown in Fig. 5, the collected acceleration values use only 80 bits of the full range (1023 bits) of the A/D converter. Passing the signal through an amplifier (e.g. OP284) before the Arduino will increase the resolution of the collected data. Students used the Arduino Nano or Uno for this project, which are both 10-bit devices. The 12-bit Arduino Due could be used to further increase resolution.

The limiting factor in the collection speed of the acceleration data is the Arduino writing to the serial port. However, the Arduino can poll an analog port much faster (at least two orders of magnitude) than it can write to the serial port. Acceleration data can be *oversampled* [20] to effectively increase the resolution of the A/D converter without reducing the sampling rate.

Finally, the PSD was performed using the coefficients directly from the DFT. Introducing windowing in the form of `pwelch` will yield better results. These simple improvements were outside the scope of this vibrations course but could be integrated to use ideas from signal processing.

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